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Different dimensional matrices in mathematics

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ABSTRACT

The "Multidimensional Matrices" is an original study of Multiform Matrices is introduced by the author in 2004 in mathematics. It generalizes the concept of the traditional matrix which is in 2 dimensions (columns and rows) and has a plan view as a rectangle to a Multiform matrix that has many dimensions and it can be described in a 2 dimensional plan or 3-dimensional space, its name indicates its form, the Multiform Matrix can have any form in the nature or any geometrical form. The introduced Multiform Matrix will have many applications in mathematics, physics, engineering and all scientific domains and especially in storing data and information on any geometrical form. The Multiform Matrix combines many matrices into a single matrix that contains the same property of all other combined matrices. The elements in the matrix can be scalars, equations, parameters, vectors, complex numbers… The main goal of introducing these kinds of matrices is to compress the information in the matrices into a very small surface and shape without losing any characteristic of the elements in the matrices and guarding the same properties of the traditional matrices such as operation between matrices and many others important properties.

Index Terms: Matrix, Multiform Matrix, Multidimensional Matrix, Mathematics, Cylindrical Matrix, Spherical Matrix.

1. INTRODUCTION

In mathematics, a matrix (plural matrices) is a rectangular array of numbers, symbols, or expressions. The individual items in a matrix are called its *elements* or *entries*. An example of a matrix with six elements is

$$\begin{bmatrix} 1 & 3 & 65 \\ 23 & -97 & 2 \end{bmatrix}$$

Matrices of the same size can be added or subtracted element by element. The rule for matrix multiplication is more complicated, and two matrices can be multiplied only when the number of columns in the first equals the number of rows in the second. A major application of matrices is to represent linear transformations, that is, generalizations of linear functions such as $f(x) = 4x$. The product of two matrices is a matrix that represents the composition of two linear transformations. Another application of matrices is in the solution of a system of linear equations. If the matrix is square, it is possible to deduce some of its properties by computing its determinant. For example, a square matrix has an inverse if and only if its determinant is not zero. Eigenvalues and eigenvectors provide insight into the geometry of linear transformations.

Matrices find applications in most scientific fields. In physics, matrices are used to study electrical circuits, optics, and quantum mechanics. In computer graphics, matrices are used to project a 3- dimensional image onto a 2-dimensional screen, and to create realistic-seeming motion. Matrix calculus generalizes classical analytical notions such as derivatives and exponentials to higher dimensions. A major branch of numerical analysis is devoted to the development of efficient algorithms for matrix computations, a subject that is centuries old and is today an expanding area of research. Matrix decomposition methods simplify computations, both theoretically and practically. Algorithms that are tailored to the structure of particular matrix structures, e.g. sparse matrices and near-diagonal matrices, expedite computations in finite element method and other computations. Infinite matrices occur in planetary theory and in atomic theory. A simple example is the matrix representing the derivative operator, which acts on the Taylor series of a function.

In this paper, introduced an original study called “Multiform Matrix” in which it generalizes the form of a rectangular matrix to any geometrical form in 1-dimension, 2-Dimensions, 3-Dimensions or multi-dimensional Matrix. The main goal of introducing these kinds of matrices is to reduce as possible the space of the matrix drawn in 1-Dimension, 2-Dimensions or multi- dimensions. For example if we have a matrix in 1-Dimension and it has 100 elements then in normal case it takes more space compared to a Spiral Matrix which is a particular case of the Multiform Matrix. And moreover, the Multiform Matrix reserves the properties of the traditional matrix, so we can add to multiform matrices which have the same form, we can subtract, multiply, divide and do any operation between multiform matrices. Also we can find other interesting properties similar to the traditional matrix such as determinant, Eigen value, eigenvector and Eigen space… of the matrix. And much more properties are conserved.

2. PYRAMID MATRIX IN 2 DIMENSIONS

A pyramid matrix in 2-Dimensions is similar to a triangular form with the top contains one element, the second row under the top contains 1+n elements, the third row contains 2n + 1 and so on…

$$\begin{bmatrix} & & & a & & & \\ & & b & & c & & \\ & d & & e & & f & \\ g & & h & & i & & j \\ & & & \dots & & & \end{bmatrix}$$

The number of elements in each row can be developed using the arithmetic development as following

$U_1=1$, is the number of elements in the first row (top row)

$U_2=U_1+n= 1+n$, is the number of elements in the second row

$U_3=U_2+n= 1+2n$, is the number of elements in the third row…

$U_m= 1+(m-1)n$, is the number of elements in the row m

With n indicates the additional number of elements in each row, and m indicates the number of the row

In the previous example we have taken n=1.

3. PYRAMID MATRIX IN 3-DIMENSIONS

A pyramid matrix in 3-Dimensions is the general case of the previous triangular pyramid in 2-Dimensions, it has a Pyramid form with a square base, and the top contains one element, the second layer under the top contains $(1+n)^2$ elements, the third layer contains $(2n+1)^2$ and so on...

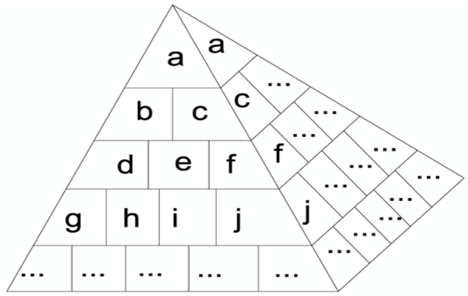

Figure 1
A Pyramid in 3-Dimensions with a square base

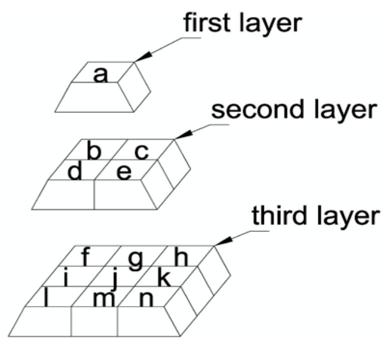

Figure 2
Different layers of a Pyramid in 3-Dimensions with a square base

The number of elements in each layer can be developed using the arithmetic development as following
$U_1 = 1$, is the number of elements in the first layer

$U_2 = (U_1 + n)^2 = (1+n)^2$, is the number of elements in the second row

$U_3 = (U_2 + n)^2 = (1+2n)^2$, is the number of elements in the third row...

$U_m = (1+(m-1)n)^2$, is the number of elements in the row m

With n indicates the additional number of elements in each row, and m indicates the number of the row. In the previous figures 1 and 2, we have taken n=1.

Each layer has a number of rows and columns. And each element is denoted by $\boldsymbol{a}_{l,r,c}$

With
a is the element in the matrix, it can be a scalar, parameter, equation, vector...

l Indicates the layer of the presented element

r indicates the row number of the presented element in the layer l
c indicates the column number of the presented element in the layer l

4. DISK MATRIX (OR FLAT CIRCULAR MATRIX) IN 2-DIMENSIONS

A “Disk Matrix” is also called “Flat Circular Matrix” or “Ring Matrix”, is a matrix that has a circular form as shown in the following figure:

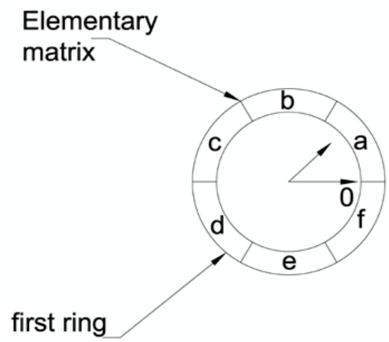

Figure 3

A Disk Matrix with one ring and 6 elements, each element is determined by an angle. The beginning of the elements of each ring has an angle equal to zero “0” as indicated on the drawing.

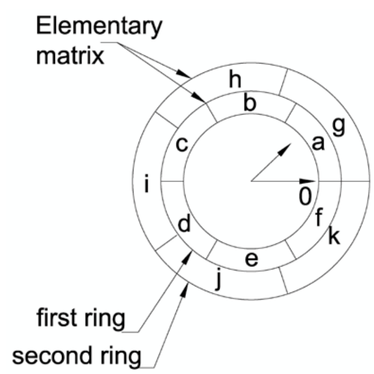

Figure 4

A Disk Matrix with two rings, the first one contains 6 elements, and the second one contains 5 elements, each element is determined by an angle and by the number of its ring. The beginning of the elements of each ring has an angle equal to zero “0” as indicated on the drawing.

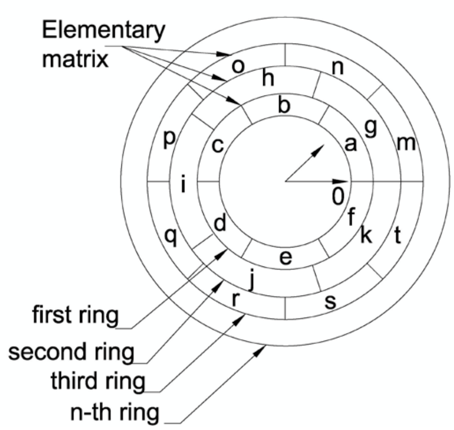

Figure 5

A Disk Matrix with “n” rings, the first one contains 6 elements, the second one contains 5elements, and and so on… each element is determined by an angle and by the number of its ring.

The beginning of the elements of each ring has an angle equal to zero "0" as indicated on the drawing.In the Disk Matrix in 2-Dimensions, the elements are distributed according to two indices, the first one is angle $(i.e.\ 10^{\circ}, 30^{\circ}\ldots)$ from the horizontal axis which is the beginning of each element in each ring. And the second one is the number of the ring.
So, an element is denoted by $a_{a,r}$

With a is the element in the matrix, it can be a scalar, parameter, equation, vector…

a indicates the angle the presented element
r indicates the ring number of the presented element in the matrix

The center of the disk is void, only the elementary matrices form rings around the center. The Disk Matrix is very important and it is the basis of the Cylindrical Matrix, and Spherical Matrix.

5. CYLINDRICAL MATRIX IN 3-DIMENSIONS

The "Cylindrical Matrix" is the vertical extension of the Disk Matrix in 3-Dimensions; the Disk Matrix is a layer in the Cylindrical Matrix.

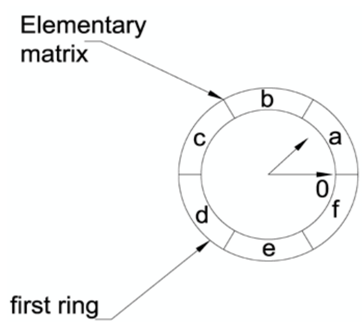

Figure 6
A Cylindrical Matrix formed by many layers of the Disk Matrix, each layer is independent from the others layers and each ring is also independent from other rings.

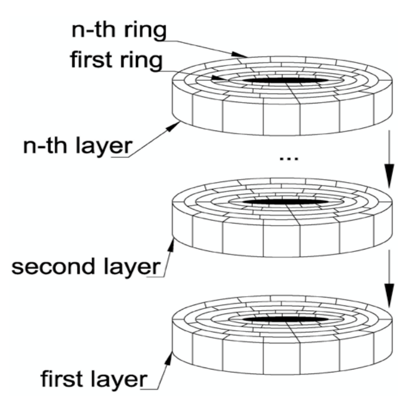

Figure 7
A closer view gives more details about how the layers are formed and how they are heaped up to form a Cylindrical Matrix

The elements in this matrix are denoted by $a_{a,r,l.}$
With
a is the element in the matrix, it can be a scalar, parameter, equation, vector…

a indicates the angle of the presented element

r indicates the ring number of the presented element in the matrix

l indicates the layer number of the presented element in the matrix

The center of the disk is void, only the elementary matrices form rings around the center.

6. SPHERICAL MATRIX IN 3-DIMENSIONS

The "Spherical Matrix" is the extension of the Disk Matrix in 3-Dimensions in a spherical shape; the Disk Matrix is a layer in the Spherical Matrix. The Spherical Matrix is formed by many layers of the Disk Matrix heaped up over each others to form a spherical shape in the space as in the following figure:

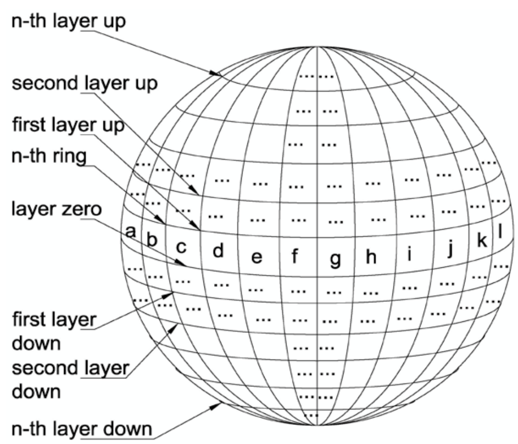

Figure 8

Spherical Matrix, it is formed by many layers of the Disk Matrix heaped up in a form of sphere

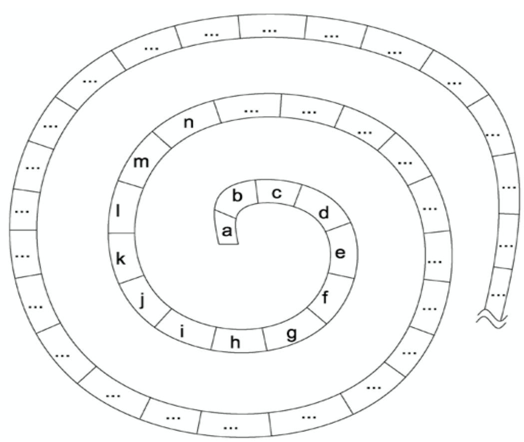

Figure 9

A closer view gives more details about how the layers are formed and how they are heaped up to form a Spherical Matrix

The elements in this matrix are denoted by $a_{a,r,l}$

With

a is the element in the matrix, it can be a scalar, parameter, equation, vector…

a indicates the angle of the presented element

r indicates the ring number of the presented element in the matrix

l indicates the layer number of the presented element in the matrix

The center of the disk is void, only the elementary matrices form rings around the center.

We can form many other matrices in 3-Dimensions or more dimensions, but in this paper, the author only introduced few examples in 3-Dimensions just to give an idea about forming multiform matrices.

7. SPIRAL MATRIX IN 2-DIMENSIONS

The Spiral Matrix in 2-Dimensions is similar to a linear matrix but it takes less space.

7.1. Spiral Matrix in 2-Dimensions

The spiral matrix is a matrix formed in 2-Dimensional space with a spiral form, it is similar to a linear matrix (1-Dimension), 2-Dimensions, or multi-dimensions but it has a spiral form in order to compress the space formed by the elements of this matrix. The form of such matrix is as following:

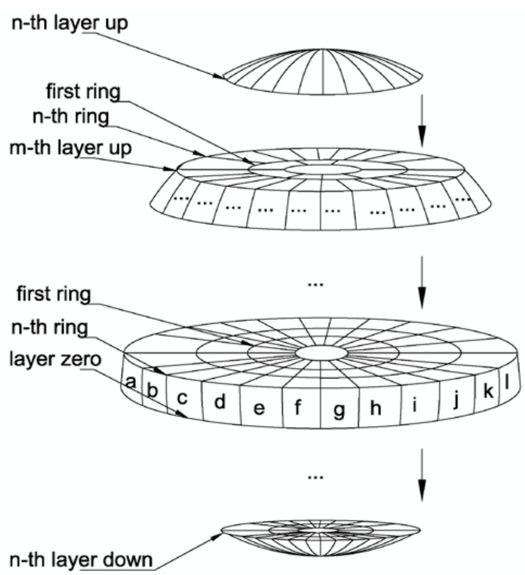

Figure 10

A Spiral Matrix with a single dimension

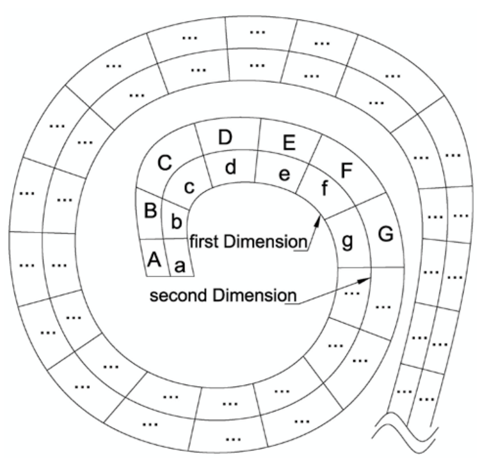

Figure 11

A Spiral Matrix with two dimensions

The elements in this matrix are denoted by $a_{p,\,d}$

With a is the element in the matrix, it can be a scalar, parameter, equation, vector…

p indicates the position order of the element in a single dimension in the matrix.

d indicates the dimension of the presented element in the matrix.

For example $a_{4,3}$ is the element presented in the fourth position in the third dimension.

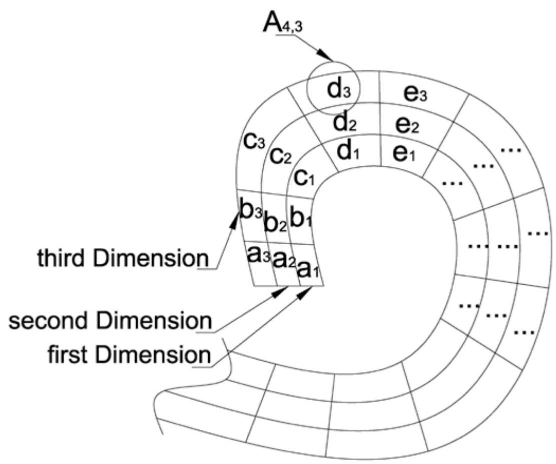

Figure 12

A Spiral Matrix with three-dimensions, the element $a_{4,3}$ is indicated by a circle

7.2. Spiral Rectangular Matrix in 2-Dimensions

The spiral Rectangular matrix is a matrix formed in 2-Dimensional space with a spiral rectangular form, it is similar to a linear matrix (1-Dimension), 2-Dimensions, or multi-dimensions but it has a spiral rectangular form in order to compress the space formed by the elements of this matrix. It is similar to the spiral matrix in the previous section. The form of such matrix is as following:

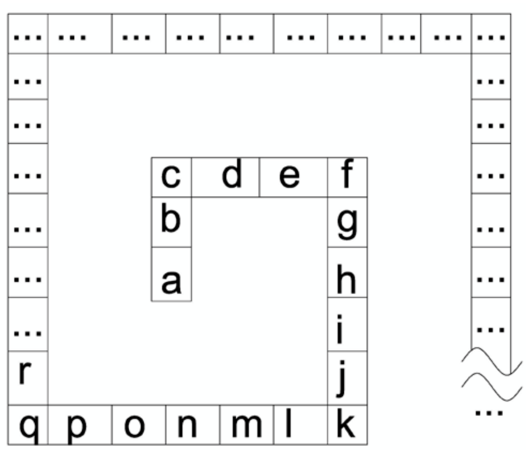

Figure 13

A Spiral Rectangular Matrix with a single dimension

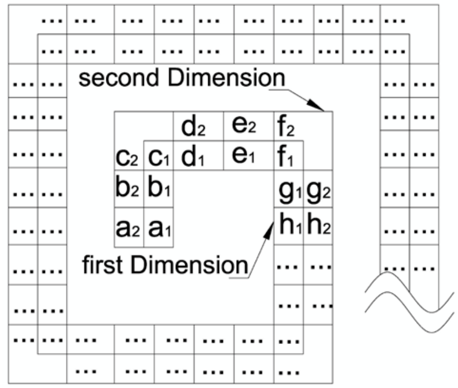

Figure 14
A Spiral Rectangular Matrix with two dimensions

The elements in this matrix are denoted by $a_{p,\, d}$

With

a is the element in the matrix, it can be a scalar, parameter, equation, vector…

p indicates the position order of the element in a single dimension in the matrix.

d indicates the dimension of the presented element in the matrix.

7.3. Spiral Triangular Matrix in 2-Dimensions

The spiral Triangular matrix is a matrix formed in 2-Dimensional space with a spiral triangular form, it is similar to a linear matrix (1-Dimension), 2-Dimensions, or multi-dimensions but it has a spiral triangular form in order to compress the space formed by the elements of this matrix. It is similar to the spiral matrix in the previous section. The form of such matrix is as following:

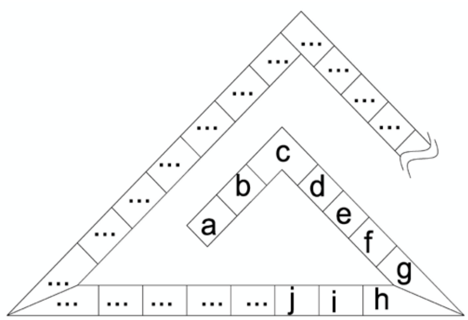

Figure 15
A Spiral Triangular Matrix with a single dimension

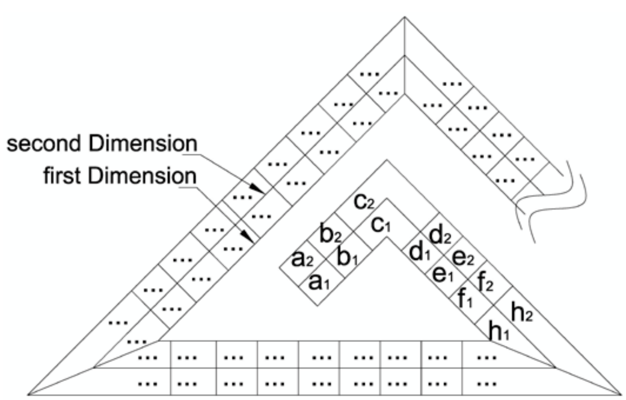

Figure 16
A Spiral Triangular Matrix with two dimensions

The elements in this matrix are denoted by $a_{p,\,d}$

With

a is the element in the matrix, it can be a scalar, parameter, equation, vector…

p indicates the position order of the element in a single dimension in the matrix.

d indicates the dimension of the presented element in the matrix.

8. SPIRAL MATRIX IN 3-DIMENSIONS

The Spiral Matrix in 3-Dimensions is similar to the Spiral Matrix in 2-Dimensions but it is extended vertically in order to form a matrix in 3-Dimensions.

8.1. Spiral Matrix in 3-Dimensions

The spiral matrix in 3-Dimensions is a matrix formed in 3-Dimensional space with a spiral form, it is similar to a matrix in 2-Dimensions, but it has a spiral form in order to compress the space formed by the elements of this matrix. The form of such matrix is as following:

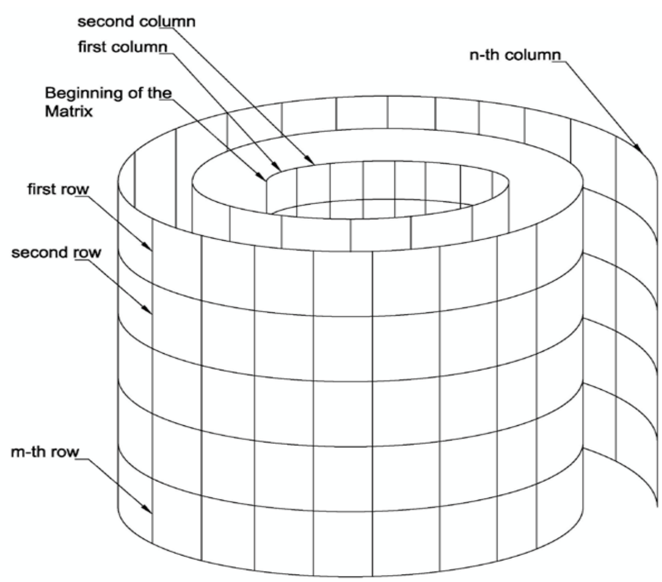

Figure 17

A Spiral Matrix in 3-Dimensions similar to a sheet of paper with a spiral form

The elements in this matrix are denoted by $a_{r,c,l}$

With

a is the element in the matrix, it can be a scalar, parameter, equation, vector…

r indicates the row of the element in the matrix.

c indicates the column of the element in the matrix.

l indicates the layer of the element in the matrix.

(In the figure 17, the matrix is formed with one layer, more layers can be added to form a 3-dimensional matrix with 3 indices to determine each element).

The same concept can be applied for Spiral Rectangular Matrix in 3-Dimensions, to the Spiral Triangular Matrix in 3-Dimensions and to any other form.

8.2. Spiral Rectangular Matrix in 3-Dimensions

The Spiral Rectangular Matrix in 3-Dimensions is a matrix formed in 3-Dimensional space with a spiral rectangular form, it is similar to a 2-dimensional matrix, or 3-dimensional matrix but it has a spiral rectangular form in order to compress the space formed by the elements of this matrix. It is similar to the spiral matrix in the previous section. The form of such matrix is as following:

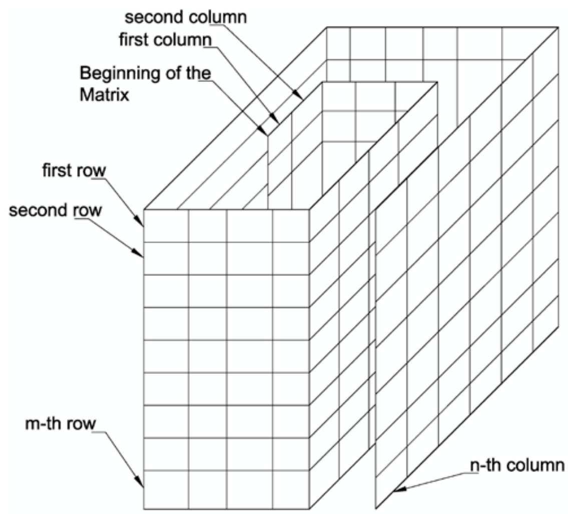

Figure 18

A Spiral Rectangular Matrix with a single dimension

The elements in this matrix are denoted by $a_{r,c,l}$

With

a is the element in the matrix, it can be a scalar, parameter, equation, vector…

r indicates the row of the element in the matrix.

c indicates the column of the element in the matrix.

l indicates the layer of the element in the matrix. (In the figure 18, the matrix is formed with one one layer, more layers can be added to form a 3-dimensional matrix with 3 indices to determine each element).

9. WHY DO WE USE MULTIFORM MATRICES?

The Multiform Matrix is similar to the traditional matrix (rectangular matrix) in all aspects, for example we can add multiform matrices, multiply them, and do any operation between similar forms of matrices, and moreover, the multiform matrices in three dimensions are more general compared to the traditional matrix and they can store more information in a smaller volume.

The utilization of the Multiform Matrices is as following:

- To store information and data on different shapes and volumes.
- We are able to store data on three dimensional objects and shapes.
- Fast operation between matrices.
- Same properties as the traditional matrix.
- Operations between matrices can be done in three dimensions.

10. CONCLUSION

In this paper, the “Multiform Matrix” is a matrix formed on different geometrical shapes in order to store information and to manipulate them similar to the traditional matrix. The main goal for introducing these kinds of matrices is to store information into a smaller matrix that has a smaller shape compared to the traditional matrix. In this paper, only few examples and few shapes are developed, but we can form matrices from any shape that conserves the property of the traditional matrix and the elements in the matrix.

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